Understanding Student Difficulties in First Year Quantum Mechanics Courses

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Introduction

What is the best approach when it comes to teaching introductory quantum mechanics? Should students first learn how to *do* quantum mechanics, by doing numerous mathematical manipulations? Or should students start by understanding what quantum mechanics means? Perhaps we should do both. Then the question remains how the *doing* and the *understanding* fit together.

To answer these questions, the learning process of students has to be closely followed. In this paper problems are presented that have been observed in two introductory courses on quantum mechanics, one on quantum physics, and one on quantum chemistry. Some of these problems have already been described in literature. These observations form a starting point for future research on the learning of quantum mechanics.

Methodology

The researcher plays an participatory role in teaching, as well as in developing the education. This is done in collaboration with the 'regular' teachers of the courses. Hopefully this method will safeguard implementation, or adoption of the findings of the research.

In the first stage of the research, the researcher participates in current practice, without pushing his views. The goal is to identify problems, or difficulties, and to develop hypotheses based on these observations. In the second stage, the researcher will develop education in collaboration with the teachers to test the hypotheses from the first stage. This paper reports on the first stage of the research[1].

The researcher had different roles during lectures and tutorial sessions. During lectures, the researcher was merely observing, sitting amongst the students and taking notes. During tutorial sessions the researcher was sometimes only observing (e.g. during plenary discussions), but most of the time he took on the role of second teacher, answering questions from students and helping them solving problems. These participatory observations enabled the researcher to experience the difficulties students have when solving problems and a teacher has when helping students. To closely monitor the process tape recordings were made during all lectures and tutorial sessions.

Results

The two courses that were observed differed in some respect. On average the Quantum Physics (QP) course was attended by 40 students. The textbook used was Griffiths (2005) (chapters 1 and 2). Only the basics of quantum theory were discussed. The Quantum

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[1] A full report can be found on the author's website: `http://staff.science.uva.nl/~lkoopman/`

Chemistry (QC) course was attended by approximately 10 students. Here Atkins and Paula (2002) (chapters 11-14) was used as textbook. In short, these chapters treat the principles of quantum theory, atomic and molecular structure. In the remainder of this section several notable observations from both courses are discussed, ordered by theme.

Quantum versus classical mechanics Classical mechanics can play various roles in quantum mechanics learning (Fischler & Lichtfeldt, 1992; Steinberg et al., 1999). Several observations related to these roles are discussed in this section.

The Schrödinger equation explicitly refers to potential energy, defining the surroundings, or environment of a particle. In a discussion with the teacher during a tutorial session two students appear to have fundamental difficulties with concepts like (total) energy and potential energy. They think it is not possible for a particle to have negative total, or potential energy. It seems that they use an implicit definition for energy, and perhaps for potential energy, that disables them from understanding the explanation of the teacher. This discussion emphasizes that the understanding of the concepts of energy and potential energy, as used in classical mechanics, is important for learning quantum mechanics. In a talk Redish et al. (1997) report on difficulties students have with energy in quantum mechanics, although they do not mention the conception that energy cannot be negative.

The teachers often compare quantum mechanical results with what is to be classically expected. For students to appreciate these comparisons, they will have to know the classical results. For example, the teacher in the QC course compares tunneling with the throwing of a ball against a wall. Classically the ball will bounce back, but quantum mechanically, the "ball" has a chance to tunnel through the "wall". A student then asks if it is possible for this particle to fly back through the hole once it is on the other side of the barrier. Using the word "hole" shows that this student does not understand what is happening: there is no hole. The comparison made by the teacher proves to be delicate.

Classical conceptions can also be a hinderance. In a discussion in the fourth week, a student in the QP course says he thinks it should be possible to give a function which describes the trajectory of a particle. He is, however, aware of Born's statistical interpretation, and he is able to give its definition. Because of this the teacher might not even notice that this student holds a deterministic view. Most of the problems that are discussed in tutorial sessions do not directly test this. But teachers will find it unsatisfactory when students cling to a classical, deterministic view. Moreover, it could well be that students have difficulties solving problems because of having a deterministic view, when an indeterministic view is needed to interpret the problem correctly.

Difficulties with Heisenberg's uncertainty relation In both courses the Heisenberg uncertainty relation is 'defined' as the impossibility to measure position and momentum simultaneously with arbitrary accuracy: measuring position (momentum) with high accuracy makes it impossible to measure momentum (position) with high accuracy. This is contrary to the definitions given in the text books. There the uncertainty relation is defined in terms of the spread in momentum and position of statistical ensembles.

When asked what the uncertainty relation is, students in the QC course repeat the teacher's definition, and add that the measurement influences the system. They feel that it should be possible to measure both position and momentum with arbitrary accuracy. When asked if it is possible to predict the position and momentum of a particle, a student answers affirmative, but he adds that it probably contradicts Heisenberg's uncertainty relation.

It is important that students understand Heisenberg's uncertainty relation, because it shows plainly that quantum theory is not deterministic. We have seen that students think the uncertainty relation is a consequence of the measurement process, something that has already been reported in literature (Styer, 1996; Johnston et al., 1998; Müller & Wiesner, 2002).

Interpretation of quantum mechanics The teacher of the QP course uses the Copenhagen interpretation. He makes a subtle distinction in the meaning of the wave function when talking about interpretation, and when conducting measurements. In the Copenhagen interpretation the wave function provides an exhaustive description of the properties of the particle: the wave function is identified with the individual particle. When performing a *measurement*, it is not possible to say anything about the wave function, or the probability distribution: multiple measurements are needed on many particles.

This distinction has consequences for the meaning of the Heisenberg uncertainty relation, as mentioned in the previous paragraph. The Heisenberg relation, as derived from the postulates of quantum mechanics, reads: $\sigma_x \sigma_p \geq \frac{\hbar}{2}$, with σ the standard deviation for respectively the position x and momentum p. In the Copenhagen interpretation it makes sense to say that σ_x and σ_p are properties of the particle itself, because the wave function holds the information for σ_x and σ_p. In the statistical ensemble interpretation (Ballentine, 1970) such a claim cannot be made: the wave function only contains information on an ensemble, and as such, σ_x and σ_p only have a meaning for the ensemble as a whole. Therefore, in the Copenhagen interpretation it follows that the Heisenberg uncertainty relation prohibits simultaneous measurement of both position and momentum with high accuracy. In the statistical ensemble interpretation, this does not follow from the uncertainty relation. There the position and momentum of individual particles can possibly be measured with arbitrary accuracy. However, an ensemble of similarly prepared systems, cannot have a spread in x and p, in such a way that the uncertainty relation is violated.

Students might benefit from explicitly discussing different interpretations of quantum mechanics, and making clear what is to be considered interpretation, and what formalism. Müller and Wiesner (2002) use the statistical ensemble interpretation in an introductory course because of its conceptual clarity.

Creating a mental picture of 'things' From the questions students ask it can be seen that they have difficulties understanding what the meaning is of certain formula's, calculations and concepts. For example, when discussing the roots in a probability distribution for a particle, a student in the QC course asks: "If you translate that to a tangible idea? Where is the chance zero... For instance: an electron *describes a wave* round the nucleus, then where is [the chance] zero?" (author's emphasis). This student explicitly asks the teacher for a more insightful explanation. From his question we can also see that he visualizes the electron in an atom incorrectly: he thinks it describes a wave. Both courses do not seem to address this problem systematically. Zollman et al. (2002) describe a hands-on approach to learning quantum mechanics, that might be useful here.

Conclusion

The observations show clearly what we already knew: quantum mechanics is a difficult subject. It is striking however that after instruction, students still seem to hold onto a deterministic world view. Also, we have the impression that students do not have consistent ideas on quantum mechanics. It is questionable whether students master the basics of quantum mechanics.

Few, or no research has been done on the influence that the various interpretations of quantum mechanics could have on learning quantum mechanics. The interpretation of quantum mechanics might be considered as a part of the language we use to interpret, or give meaning to the formalism of quantum theory. In that sense, the following question might be interesting: "How does the language, used by teachers and students in first year quantum mechanics courses, affect students' learning of quantum mechanics?". A more concrete sub-question is: "What impact does the use of the Statistical Interpretation of quantum mechanics have on the understanding of quantum mechanics by first year physics/chemistry students?". Furthermore it is questionable if the approach used by Griffiths (2005) to first teach students to do quantum mechanics, and then teach them what it means, is successful.

For the next stage of the research, small changes will have to be made to the courses as they are given currently. It is therefore important to have full cooperation from the teachers of both courses. This will be a challenge for the next couple of months. Discussing subjects like the interpretation of quantum mechanics, can be a delicate subject. We do not want to fall in to a discussion of what is the right, and what is the wrong interpretation. This is an ongoing debate, that is not the subject of this research.

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