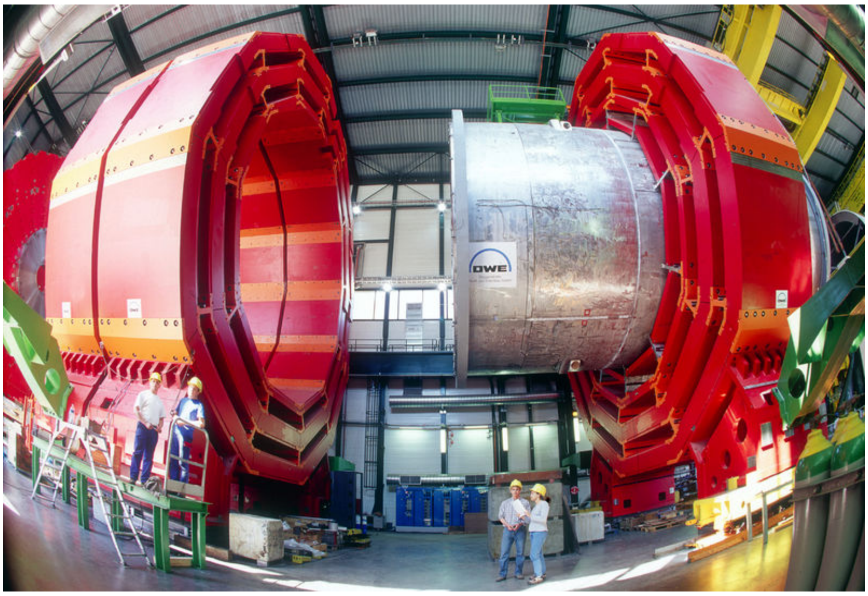


Compact Muon Solonoid (CMS) detector tasked with discovering the Higgs particle which can decay into 4 muons.

Dick Hoekzema and Ed van den Berg

**Project Modern Physics**
**Centre for Science and Mathematics Education**
**Utrecht University**
**Utrecht, Netherlands**

Papers and supporting materials presented at the 1st European Physics Education Conference organized by the European Physical Society Division of Education, Bad Honnef, Germany, 4 – 7 July 2005.

# Contents



**Comment**

**The exam problems were developed by a team of teachers.**

# Modern Physics in Secondary Schools in the Netherlands[1]

Ed van den Berg, Dick Hoekzema
Centre for Science Education
Utrecht University, Utrecht
Netherlands

## Overview

The Dutch Modern Physics Project was initiated due to a general feeling that modern physics was poorly represented in the 1998 Dutch curriculum for the upper secondary pre-university science and technology stream. Earlier creative attempts to introduce quantum physics and elementary particles did not catch on and did not get beyond the trial state. From these attempts we learned that we had to be modest. In order to evade the mathematical difficulties of the Schrödinger equation we merely use the limited particle/wave-in-box model. However, we discovered that this model has surprisingly many interesting applications. In particle physics, we evaded the conceptual difficulties of interpreting Feynman diagrams as a way of understanding interactions, by using only very much simplified reaction diagrams. However, even these simplified diagrams provide a powerful tool for focusing on conservation laws, symmetries and elementary particle reactions rather than on the multitude of particles. The reaction diagrams caught on and allowed students to judge which reactions are possible and predict other reactions by applying symmetries.
Our lesson materials require about 40 lessons. The number of schools adopting the program has increased steadily and will be about 40 in 2005-2006. At the poster session we like to exchange ideas with other participants about the content and teaching approach of modern physics courses at the secondary level. Selected parts of our lesson materials will be available in English.

The **home base** for the Dutch Project Modern Physics is the Centre for Science and Mathematics Education in the Department of Physics and Astronomy of the University of Utrecht. The Centre is the leading Centre for Science Education Research and Development in the Netherlands.

**Background**: In 1996 a new national curriculum was formulated for the pre-university stream of secondary education in the Netherlands. The Science Committee felt that Modern Physics did not get sufficient attention and requested a special project to explore alternatives and develop lesson materials for teaching and learning Modern Physics.

During the period 1996 – 2002 lesson materials were developed and tested through several cycles of piloting in schools and revision. Currently the course consists of a package of materials for about 40 lessons.

The **objectives** for the Modern Physics package are that it should:

- give a valid impression of present day Physics;
- be conceptually interesting and challenging but mathematically limited;
- provide opportunities for in-depth study as well as for bridges to applications;
- be testable in National Secondary School Exams;
- be interesting to Physics teachers and be "teachable".

A special **characteristic** of the materials, which is immediately noticed by the students, is the representation of Physics as a subject that is very much in development, with many uncertainties in interpretation, even in topics where predictions and measurements can have a high degree of accuracy.

[1] Poster presented at the first ECEP Physics Education Conference, Bad Honnef (Germany), 4-7 July 2005.

The Modern Physics materials were written to articulate these differences in interpretation. What are electrons? How do you picture an atom? Can it be pictured? What is a photon? To what extent are particles *wave-like* or waves *particle-like*, etc. In other words, we have not taken the "shut-up and calculate" view of quantum physics, but a blend of a conceptual and computation approach. As a consequence quantum physics is very much presented as a work in progress with an exotic smell. Students sense that within the first 4 lessons. Nature turns out to have an exotic side and students find that very interesting. They get a rather different view of physics. Some of the weaker students have difficulties with the uncertainties in interpretation; they would rather have the straightforward classical physics. Most students do like the exotic side. They make statements such as the following:

> *Physics even has connections with Philosophy*
>
> *We learn things in our physics lesson that nobody else in school knows about*,
>
> *The others in school know about atoms and molecules, and protons and neutrons, but they do not know about gluons and quarks and pions, only we know that* or
>
> C*lassical Physics is comprehensible but can be boring, Modern Physics is difficult but exciting*

Another characteristic is the thinking in terms of simple quantitative models, and evaluating their benefits and limitations. For example, in the atomic physics chapter the students learn about the particle-in-a-box model and about possible refinements; in astrophysics students use simple secondary school physics to investigate alternatives for the energy generation in the Sun, such as burning or gravitational contraction and then they evaluate the plausibility of these models.

**Topics are**:

- What is Modern Physics?
- Photons and electrons, wave-particle dualism, probability;
- Atoms and molecules; particle-in-a-box model and applications of the model to explain spectra, strength of materials, and colors of pigments, and other phenomena;
- Reactions of atoms, nuclei, and elementary particles; conservation laws and symmetries;
- Astrophysics with an emphasis on the use of models and on some main observational methods, like the use of spectra for determining temperatures, compositions and velocities.

**Time required**: Most participating teachers devote about 40 lessons of 45 – 50 minutes to Modern Physics. That is almost double the time spent in non-project schools on modern physics, excluding radioactivity, which is elsewhere in the curriculum. The total number of lessons for physics in grades 10 – 12 is about 370 so even 40 lessons use only slightly over 10% of curriculum time.

**Target population**: The top 10% students of Dutch secondary schools who may be inclined to choose careers in Engineering or Science.

**Grade level**: 12

**Participating schools, teachers, and students:** Table 1 shows the numbers in the project. The total of about 500 students in 2005/2006 is about 8% of the national target population.

Table 1: Participating schools and students

| | 2001/2002 | 2002/2003 | 2003/2004 | 2004/2005 | 2005/2006 |
|---|---|---|---|---|---|
| Schools | 11 | 13 | 24 | 34 | 39 |
| Teachers | 11 | 13 | 27 | 40 | 50 |
| Students | 143 | 184 | 322 | 446 | >500 |
| Schools using part of the material only | | | | | >4 |

**Teaching methods**: Typical lessons of participating teachers consist of short plenary 10 – 15 minute introductions to the theory or plenary discussions of assignments followed by student activities such as small group discussions of conceptual questions, solving problems, doing computer simulations or working with applets, searching for additional information on internet or CDs, etc. For some topics the project has developed fast-feedback worksheets, which can be used to quickly diagnose student errors and provide immediate feedback for both teacher and student. Recently the project developed a long list of conceptual questions in order to exercise the conceptual aspects of modern physics.

**Examinations**: A special objective for the project was to proof that also modern physics can be tested under the typical Dutch examination system. A large collection of examination problems was developed and each year a special version of the national examination is produced for project schools. Examination results of project schools have been comparable to those of other schools. In fact, project students score better on the joint classical physics part of the exam.

**Excursions**: Most students/schools participate in annual excursions to one of the big European Physics laboratories such as the accelerators at CERN in Geneva and DESY inHamburg, the joint European Torus (JET) nuclear fusion in UK, the GSI heavy ion accelerator in Darmstadt Germany, and the Institute Laue-Langevin with the world's most intense neutron source in Grenoble, France. In 2004/2005 about 300 students participated in an excursion to one of these different destinations. Students pay for the costs of these excursions except for the organizational overhead. Prices vary depending on destination from Euro 160 – 215.

**Outreach**: The project also conducts outreach activities with regard to the teaching of modern physics through national workshops and seminars for teachers who are not directly involved in the project. The project has produced several sample modules, which can be used to assist teachers who are not participating in the project but would like to make use of selected parts of the lesson materials. One of these modules is a series of three lessons about conservation laws, symmetries and elementary particles.

**Future Development**: When the project started in 1998 it was conceived as a national pilot for modern physics materials, which would then be included in the national examination program. Now that the project has produced materials which are challenging but do match the ability and interest of the students and which can be examined in the Dutch examination system, big changes happen to be proposed for the Dutch education system. The most likely future for our modern physics package of lesson materials is that it will remain an option, which schools can choose rather than a compulsory part of the curriculum.

**Upper Secondary Physics Education in the Netherlands**: The Dutch secondary system consists of 3 main streams: pre-vocational and vocational education for about 60% of the cohort, general secondary education (called havo) which gives access to 2nd tier Higher Education such as that provided in Polytechnics and involves about 20% of the cohort, and pre-university secondary education (called vwo) which prepares for university level studies, another 20% of the cohort. Students in the latter group can choose among 4 streams, two of which concern science. Students in the health and science stream make up about 30% of the vwo students thus 6% of the national cohort. They take the science core subjects while students in the technology and science stream take this core plus more mathematics, physics, and chemistry. This latter group makes up about 15% of the vwo students thus 3% of the national cohort and this is the target audience for our materials.

# Particle waves in boxes in Dutch secondary schools

*Dick Hoekzema, Ed van den Berg*
*Centre for Science Education*
*Utrecht University, Utrecht*
*Netherlands*
d.j.hoekzema@phys.uu.nl, edberg51@planet.nl

## Abstract

The combination of mathematical and conceptual difficulties makes teaching quantum physics at secondary schools a precarious undertaking. With many of the conceptual difficulties being unavoidable, simplifying the mathematics becomes top priority.

The particle (wave!)-in-a-box provides a teaching model which includes many aspects of serious quantum physics, while avoiding most of the mathematics. In a Dutch quantum physics project for secondary schools this model was adopted to play a key role, also because, much to our surprise, we ran into many more applications than we originally expected. In many instances the model yields order of magnitude estimates, for instance of atomic and nuclear size, or qualitative insights, for instance about energy levels, molecular bonding and electron pressure. Moreover, either directly or with minor modifications, the particle-in-box provides reasonable approximations of a range of phenomena, including the absorption spectra of organic pigments, the mass of the proton and the spectra of quantum dots.
In our presentation we shall present some examples, and discuss a few pitfalls and potential misconceptions. Furthermore, we shall examine a few exam problems and data about the level of understanding achieved by Dutch students.

## Introduction

The particle/wave-in-box model is a fixed part of every introduction to quantum mechanics. You will surely remember. But what exactly was the role of this particle/wave-in-box model? Most of us may have forgotten. Wasn't it a didactical trick, just for education purposes? Something a bit easy before getting to the Schrödinger equation? It is difficult to believe that this simple model could be more than that. Doesn't one have to at least use a Schrödinger equation to get decent order of magnitude estimations and even then a realistic potential should be used and not a square well with infinite depth?

Our Modern Physics Project provides a package of 35 – 40 one period lessons for 12th grade High School students who specialize in science. In our project we do not include the Schrödinger equation as we think it is a bridge too far. However, apart from emphasizing the conceptual side of quantum physics, we did want to include some computation and decided to use the particle-in-box model for that. That choice has turned out to be a lucky one as we encountered more and more phenomena where the model results in reasonable estimates. Initially we used the box model to make order of magnitude estimates for the size of atoms, nuclei, and elementary particles and to link the color of pigments such as carotene to the length of molecules. There are more possibilities. The model can provide qualitative insights into covalent bonding (why is it advantageous for atoms to share electrons); it is possible to make good estimates for the strength of materials; the box model can explain phenomena in star nuclei such as supernovas. Sometimes the model is used for qualitative insight and order of magnitude estimates. Sometimes it is used for computations with quite good results such as computing the elasticity of diamond and estimating the mass of excited states of the proton (see section V). Even some nanoscience phenomena such as quantum dots can be approximated quite well by the particle-in-box model. In short, the very simple model does have potential, not only as an educational device to explore quantum mechanics concepts, but also as a tool for first estimates.

Meanwhile experience in 30 schools has shown that students can handle the model and we think results can improve yet as teachers get more experience in teaching about quantum phenomena and using the model.

In this paper we provide a brief introduction to the model in sections I and II. In section III we show a case where the predictions do not fit very well: the hydrogen spectrum. Then we illustrate its feasibility in estimations and computations in sections IV-VI. The complete text of our lesson materials can be found on a website: www.phys.uu.nl/~wwwpmn , choose part 3b and subsequent sections, but unfortunately the website is only in Dutch language.

## I. Spectra and energy levels

When excited, low pressure gases exhibit a line spectrum. Cool gases under low pressure exhibit an absorption spectrum consisting of dark lines when exposed to white light (Fraunhofer spectrum). From these emission and absorption spectra we can construct energy levels such as that for the electron in the hydrogen atom (figure 2a). The speeds and energies of macroscopic objects such as soccer balls, bicycles, and airplanes vary on a continuous scale. A soccer ball can have a speed of 10.1 m/s but also 10.11 of 10.111. A continuous variation is possible. However, electrons can only have discrete energy values and we call this quantization. One hundred years ago this was considered strange. How is that possible? Where in nature do we find quantization? Oscillations of a string such as with a guitar. Only certain frequencies and their harmonics are possible. Quantization is nothing new and in fact was known since Pythagoras (Bunge, 2003) and it has to do with waves. This way we get to matter waves. In fact, students already encountered matter waves in a previous chapter when studying wave properties of matter including electron diffraction and double slit particle interference. Students already encountered the traveling particle waves of unbound particles. At this point in the introduction we show the standing waves on a slinky again, even if these have been demonstrated before.

## II. The particle in a box

A string resonates only at certain frequencies. Standing waves are obtained by “locking up” traveling waves, which, in the case of a string is done by clamping both ends. The interference of moving waves reflected at both ends results in a standing wave. Standing waves occur in every system in which waves are “locked up”.
The electron in an atom is locked up in this sense. If electrons in atoms behave as waves, then these should be standing waves and this should be the case for any quantum wave in a closed system, **that is for every *bound* particle.**
Going back to the string: when a string of length L is clamped on both sides, it can accommodate a half sine, a sine (first harmonic), 1½ sine (2nd harmonic) or *n* x ½ sine (figure 2). We can write the following relationship between wavelength $\lambda$ and string length $L$:

$$\lambda = \frac{2}{n} L \qquad (1)$$

Now, if we lock a (quantum) particle in a one-dimensional box, we must use a wave to describe the particle and obtain one half sine, a sine, 1 ½ sine, just like the standing waves on the string. We assign meaning to the wave by assuming that the square of the amplitude expresses the probability(-density) to encounter the particle at a particular place along the x-axis. In our lesson materials we connect this to the double slit particle interference experiments.

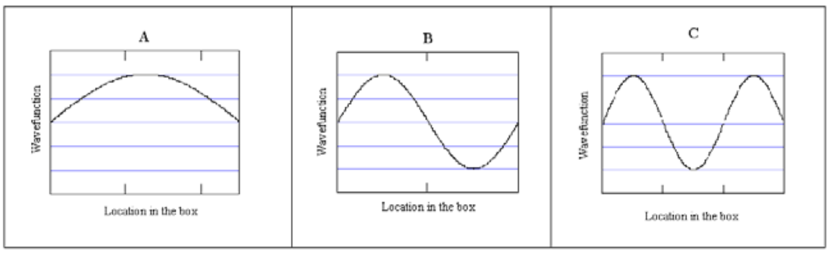



*Figure 1 Oscillations of a string, wave functions of a particle in a box.*

In exercises with diagrams like figure 1 we and ask students to indicate the most probable locations to encounter the particle. Often there is an inclination to indicate the nodes “because the particle has to pass that point every time”. This is a conceptual error, which has to be corrected by doing some exercises in interpreting various graphs of wave functions. Then we derive an expression for the kinetic energy of a particle in a box by using the relationships between momentum and wavelength and kinetic energy.

$$p = \frac{h}{\lambda} \quad (1) \qquad \text{and } E_{kin} = \frac{1}{2}mv^2 = \frac{1}{2m}(mv)^2 = \frac{p^2}{2m} \qquad \text{so } E_{kin} = \frac{h^2}{2m\lambda^2} \quad (2)$$

If we now use formula 1 to express $\lambda$ in $L$ then we obtain the well-known formula:

$$E_{kin,n} = \frac{n^2h^2}{8mL^2} \qquad (3)$$

in which $n$ = 1 indicates the ground state, $n$ = 2 the first excited state, etc.

What did we achieve? Quantization! We now have discrete energy states for the electron, numbered by *n, which* is called a quantum number, as it is typical for quantization, one of the core characteristics of quantum physics. If we express energy in units of $\frac{h^2}{8mL^2}$ then we obtain 1, 4 (= $2^2$), 9, 16, etc.
Please notice, quantization applies in the case of bound particles, not free particles. In the Schrödingerrepresentation the particle-in-box approach is equivalent to using an infinite square well potential.

## III. Comparison of atomic and particle-in-box energy levels

Figure 2 compares the energy levels determined from the Hydrogen spectrum with levels computed with the particle-in-box model. Both are discrete thus quantized. However, the particle-in-box levels are farther apart for larger quantum numbers while the observed levels are closer and closer. The particle/wave-in-box model can provide discrete levels but not the correct ones. Yet there are situations where the model works quite well.

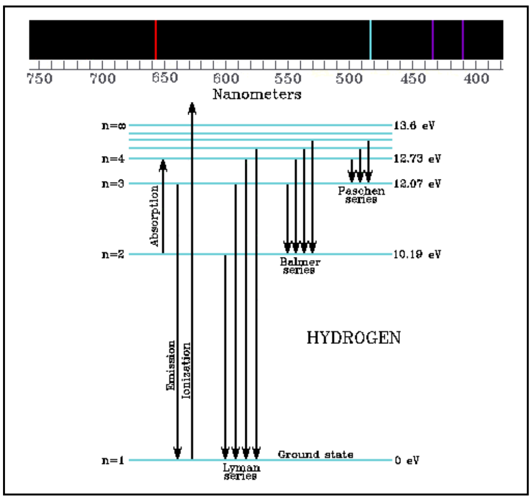
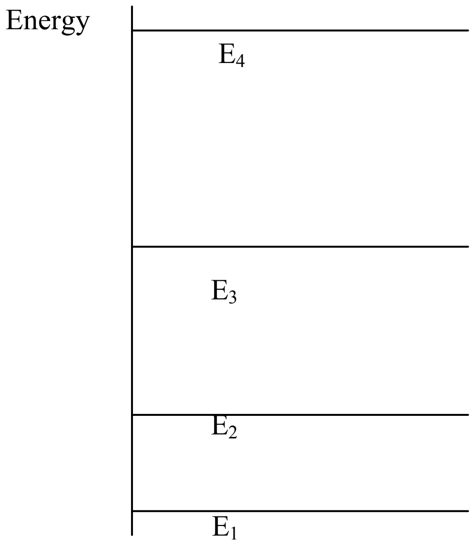



*Figure 2a Energy levels Hydrogen*




*Figuur 2b Energy levels particle/wave-in-box model*

## IV. Qualitative examples

What happens when molecules are formed? Think of two Hydrogen atoms, which together form $H_2$.

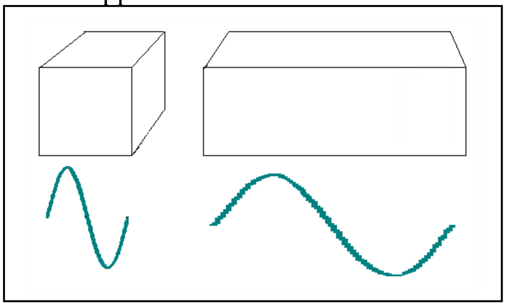


*Figure 3 Small and big box, so small and big wavelength and high and low energy.*

Compare the Hydrogen atoms with the boxes in figure 3. To the left is the small box: one Hydrogen atom. To the right is a large box: the $H_2$ molecule. Why is the $H_2$ molecule favored energetically over two H atoms? Because both electrons have a larger box available. A larger box means a larger wavelength, a smaller momentum (formula 1), and a smaller kinetic energy (formula 3). We now run into the limits to generalization. Shouldn't the same reasoning apply to Helium? No it does not. We can save the reasoning with Pauli's principle, the 2 x 2 electrons do not fit in the same ground state, but even then we obtain positive binding energies instead of negative ones. In one school students presented small projects on quantum physics. Two students computed the binding energies for noble gases hoping that those would come out smaller than for surrounding elements in the periodic table. They did not succeed, but it was educational nevertheless.

*Figure 4 Students present about quantum mechanics.*

In textbooks we read that metals have "free" electrons which are bound to the lattice rather than to individual atoms. The box of these electrons is the lattice (large box) rather than the atom. We could apply the same reasoning as with Hydrogen, so a larger box thus lower kinetic energy thus a binding energy. At the level of students this reasoning can only be used in a qualitative way but it cannot be used to compute the typical binding energy range of 1 – 3 eV. However, if we fill the particle box's energy levels until the Fermi level (so high quantum numbers) then the box model turns out to provide quite reasonable values for binding energies.

Differences in energy levels in an atom are typically several electronvolts. Differences between energy levels in the atomic nucleus are several MeV. Alpha particles which tunnel out of the nucleus or gamma photons have energies of several MeV. Can we explain this with the box model? The typical diameter of an atom is $10^{-10}$ m while the diameter of a nucleus is $10^{-15}$ m. The energy difference between a $n$=1 and $n$=2 level in a box atom is:

$$E_2 - E_1 = \frac{2^2 h^2}{8m_e L^2} - \frac{1^2 h^2}{8m_e L^2} = 3x10^{20}\,\frac{h^2}{8m_e} \qquad (4)$$

The energy difference between a $n$=1 and $n$=2 level in a box nucleus is:

$$E_{kin} = \frac{3h^2}{8m_p L^2} = 3x10^{30}\,\frac{h^2}{8m_p} = 1{,}6.10^{27}\,\frac{h^2}{8m_e} \qquad (5)$$

Please notice that in the last step we replaced $m_p$ with $m_e$. The difference between (5) and (4) is a factor $5x10^6$. Knowing that the diameter of a nucleus is several femtometer rather than 1 femtometer, the result would still be better. Isn't that nice?

In another semi-quantitative example we compute to what energy electrons will need to be accelerated in order to expose structures smaller than the nucleus. Think of the famous experiments with the Stanford linear accelerator, which provided evidence for the existence of quarks. Electrons then must have a wavelength that would be at least an order of magnitude smaller than that of the proton, for example $10^{-16}$ m. Using formula 3 we obtain an energy of 3,8x$10^4$ GeV. Yet the experiments were done with 20 – 30 GeV and that was enough to show inelastic scattering by quarks. Are we then already hitting the boundaries of the model? No, not yet. Our problem concerns relativistic energies. Instead of $E_k=½\, mv^2$ we should use $E_k=pc$. If we then use $p=h/\lambda$ we obtain:

$$E_{kin,n} = \frac{nhc}{2L} \qquad (5)$$

If we apply this with $L=10^{-16}$ m and $n$=1 then $E_{kin}$ = 2,0x$10^{-11}$ J = 6,2 GeV, which is a reasonable order of magnitude. By using electrons of 20 GeV we look at a distance scale of 1/3 x $10^{-16}$ m.

## V. Quantitative examples: the proton

The last examples were already quantitative. Let's proceed in this direction by doing some computations with a proton and its quarks. A proton consists of 3 quarks: 2 up quarks (u) and 1 down quark (d). The up quarks have a mass of 3 MeV/$c^2$ and the down quark has a mass of 6 MeV/$c^2$. Yet the proton mass is 938 MeV. The total mass of the 3 quarks comes to 12 MeV. The rest would have to be kinetic energy of the quarks. Let's assume this to be true. Then $E_k$ = 926 MeV which is 1,5x$10^{-10}$ J. Knowing the energy, we could compute the size of the proton.

$$E_{kin} = \frac{hc}{2L} + \frac{hc}{2L} + \frac{hc}{2L} = \frac{3hc}{2L} = 1,5x10^{-10}\,J$$ for the three quarks together

Hence we find $L$=1,9 x $10^{-15}$ m. Yet another interesting result!

We can go still further. There are particles with the same quark composition as the proton but with energies of 1200 MeV/$c^2$ and up which are interpreted as excited states of the proton. Let's try this. Assume that 2 quarks are in an $n$=1 state and the third in state $n$=2 in the box model. Might this explain the mass?

$$E_{kin} = \frac{hc}{2L} + \frac{hc}{2L} + \frac{2hc}{2L} = \frac{4hc}{2L} = \frac{4}{3}x1,5x10^{-10}\,J = \frac{4}{3}x926MeV = 1235MeV$$

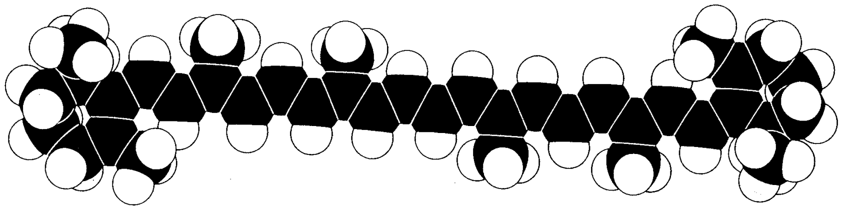
When we add the mass of the individual quarks (12 MeV) we obtain 1247 MeV. This is surprisingly close to the mass of the $\Delta^+$ particle which is composed of uud, just like the proton and has a mass of 1232 MeV/$c^2$. A pretty result, even though it is physically flawed, because the $\Delta^+$ is a particle which is different from the proton because of a different alignment the quark spins. This may remind us of the fact that quantitative results from simple models should not be trusted at face value. Nevertjelsess, at slightly higher energies (1440 MeV and up) there are other excited states of the proton quarks which *do* correspond to excited states of the quarks, so the model is not that bad after all.

## VI. Colors of pigment molecules with long carbon chains.

We present this part in the form of an examination problem, which was accomplished by students with reasonable success.

Carotene is a substance which occurs in carrots and mango's. It is a carbonhydrate with molecular formula $C_{40}H_{56}$. A model of a carotene molecule is presented in figure 5.

*Figuur 5 Carotene molecule*



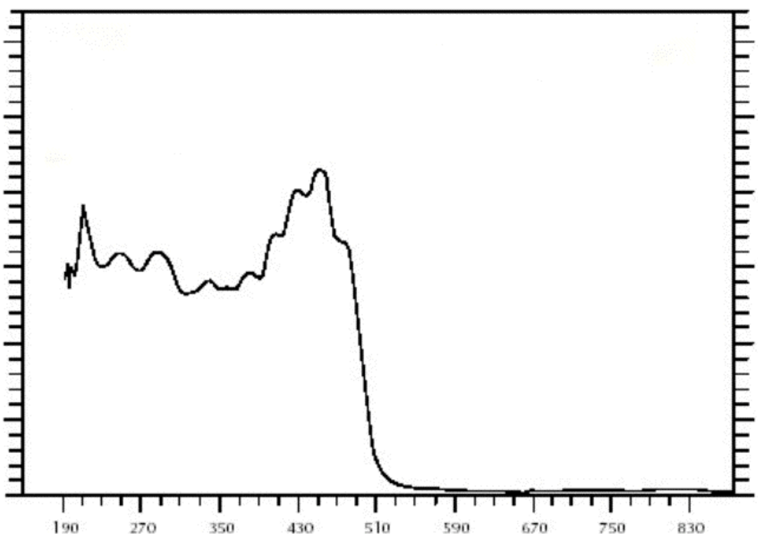
According to the theory of molecular bonding there are two kinds electron bonds[2] in carotene. The σ-electrons are localized at fixed places between the Carbon atoms. The π-electrons can move along the entire length of the molecule. The π-electrons can be described by a 1-dimensional particle-in-box model. Carbonhydrates are formed through the formation of covalent bonds between carbon and hydrogen atoms. Covalent bonds are formed because the total energy decreases when electrons are shared.

**2p** 1 □ *Explain how the sharing of electrons leads to a decrease in total energy.*

**2p** 2 □ *Explain why helium atoms , as opposed to hydrogen atoms, do not form covalent bonds.*

Figure 6 shows the absorption spectrum of carotene. Wavelength is pictured along the horizontal axis and the degree of absorption is along the vertical axis.

*Figure 6 Absorption spectrum carotene*




**3p** 3 □ *Explain the color of carotene using this spectrum.*

Carotene has 22 π-electrons per molecule. The first excitation requires an energy of 2,76 eV.

**3p** 4 □ *Use the Pauli principle to to explain which energy levels are involved in the excitation of the first π electron.*

**4p** 5 □ *Calculate the effective length along which the π-electrons can move according to this model.*

**Answers**:

[2] For more information see General Chemistry texts such as Brown et al (2000).

1. When atoms share electrons, then each electron has a larger space available. Therefore the wavelength increases so the kinetic energy per electron decreases.
2. Helium has two electrons in the ground state. According to Pauli's exclusion principle other electrons will have to go to a higher energy level. That costs so much energy that sharing does not lead to energy decrease so bonding is not advantageous in terms of energy and does not occur.
3. A large portion of the light with wavelengths below ~450 nm is absorbed, the blue color is filtered out. Longer wavelengths are reflected: red through green. This mix of colors is perceived as orange.
4. According to Pauli's exclusion principle each level fits 2 electrons. If there are 22 π-electrons, then 11 energy levels are fully occupied. The minimum energy to excite an electron then pertains to the transition from level $n$ = 11 to $n$ = 12.
5. The energy according to the particle in a box model is: $E = \frac{n^2 h^2}{8mL^2}$

   For the energy transition from n=11 to n=12 we can write:

   $$\Delta E = 2,76 \text{ eV} = 4,278 \cdot 10^{-19} \text{ J} \quad \text{and} \quad \Delta E = (12^2 - 11^2)\frac{h^2}{8mL^2}$$

   So:

   $$L = \sqrt{\frac{(144-121)(6,626 \cdot 10^{-34})^2}{8 \times 9.109 \cdot 10^{-31} \times 4,278 \cdot 10^{-19}}} = 1,80 \cdot 10^{-9} \text{ m=1,80 nm}$$

These are again reasonable results compared the 2.4 nm length of the "antenne" part of the molecule, the part with the π-bonds. The particle/wave-in-box model does allow for computations at the level of our 12th grade students and yet leads to reasonable results in a wide range of phenomena. Of course one has to watch out that application of the model does not decay into applying tricks without understanding. We try to ask mix computations with conceptual questions. In the lesson materials we also pay ample attention to the wave nature of matter and to probability interpretations.

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## Using Fast Feedback in Teaching Modern Physics[3]

Ed van den Berg, Dick Hoekzema
Centre for Science and Mathematics Education
University of Utrecht, Utrecht, Netherlands
edberg51@planet.nl and d.j.hoekzema@phys.uu.nl

Fast feedback methods combine diagnostic assessment and interactive teaching in real-time in the classroom. Using worksheets with graphical answer formats, teachers can very quickly diagnose the understanding of *many* students *during* the teaching and they can even squeeze in a 30-second interview to explore an unexpected student answer. Meanwhile the typical variation in answers to these diagnostic items leads to conceptual discussions between students, kick-starting peer-teaching. Based on quick diagnosis, the teacher adjusts pace and presentation to the learning bottlenecks encountered in the diagnosis. Students receive immediate feedback to guide their learning and they are inspired by a teacher who pays serious attention to their individual interpretations. We will present examples from an introductory modern physics course which has been adopted by over 40 secondary schools in the Netherlands. The examples are about particle theories in classical physics, wave-particle dualism, probability density, wave functions, and application of symmetries to particle reactions, and have been piloted in Dutch secondary schools as well as in Philippine college physics courses for teachers.

# Fast feedback

If we learned anything about learning from the misconception research of the 1980s, it is that there has to be a continuous interaction between teacher and students to check on students' conceptual progress (or lack thereof) and to provide constructive feedback[1]. The importance of feedback also comes from a completely different line of studies. Black and William[2] analyzed hundreds of studies on formative evaluation and concluded that formative feedback, -that is constructive reactions to student work- is one of the most powerful tools in teaching and learning, particularly if it is *not* graded (*no* marks given). Indeed, interaction and feedback are the consistent ingredients of the interactive-engagement methods promoted by PER researchers [3,4,5]. Now the problem: how can teachers provide this feedback without spending all evenings until midnight checking student work and writing in feedback comments? The answer is the use of student responses in graphical form combined with fast feedback by the teacher.

Fast feedback is a whole class method in which students work individually or in pairs but all at the same pace through a series of questions, which require answers in the form of a sketch, a graph, or a drawing. The questions are given one by one. After each question, the teacher walks around and looks at student work, asks a question here and there. After completing the question, students compare and discuss answers. When most are finished, the teacher returns to the front and discusses the one or two most frequent errors based on the work (s)he just saw in the classroom and launches the next question. It is important to keep up the pace. A question and the individual student work could take 2 or 3 minutes. The plenary discussion might take 1 or 2 minutes and then: next question. The fast feedback method works well in topics where students are known to hold strong misconceptions such as forces (force diagrams), kinematics (graphs), and electric circuits. Over a series of 6 – 8 questions, progress is very visible. At any time in the lesson the teacher will have a good idea what students understand and what not yet and further teaching is based on that information. It is necessary that the teacher keeps going around and bases the short plenary discussions on actual observations of student work or even short interviews with students while they are working. That way the teacher gets immediate feedback on what students do and do not understand, while students get feedback on their actual work. The fast feedback method can be used with any topic in physics, which allows responses in the form

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[3] Paper presented at the first ECEP conference on Physics Education, Bad Honnef (Germany), 4 – 7 July 2005

of graphs, diagrams, sketches, drawings or other short answer formats. For example, force diagrams, optics diagrams, graphs in any branch of physics, etc. A worked out example of using fast feedback in kinematics (and combined with role-play) can be found in Berg et al[6]. An overview of fast feedback and possibilities in different branches of Physics and Chemistry is contained in Berg[7]. Mazur's peer teaching methods[4,5, 8] are in essence fast feedback methods. In Mazur's method a new concept is introduced in a mini-lecture, which is then followed by a multiple-choice question. Responses can be tallied quickly through use of cards or a pushbutton system. If the tally indicates serious problems in understanding, the class will discuss the multiple-choice problems in small group or peer discussions while the lecturer listens around and interacts. On the other hand, if most students answer correctly, the lecturer proceeds with the next mini-lecture.
In the remainder of this paper article we provide two complete examples of using fast feedback in teaching modern physics. The first example is about the concepts of probability and probability density in a prelude to wave functions. The second example concerns the application of conservation laws and symmetries to elementary particle reactions to predict new possibilities from given reactions.

# Example I: Probability and quantum physics

Students, Einstein[4] and others, have problems with the probability[5] aspects of quantum physics. There are two kinds of problems:

1. using probabilities and getting used to the idea that probability plays a role in Physics;
2. using and interpreting wave functions and accompanying phenomena such as interference.

For students these problems can mix and confuse. One way to do something about it is to clearly separate the different problems and find experiences and exercises to match. Whenever there are problems in understanding, it is important to build in ample opportunities for *interaction* and *feedback*. That can be done realistically with big classes when we use graphical presentations with so called fast feedback methods. Read on!

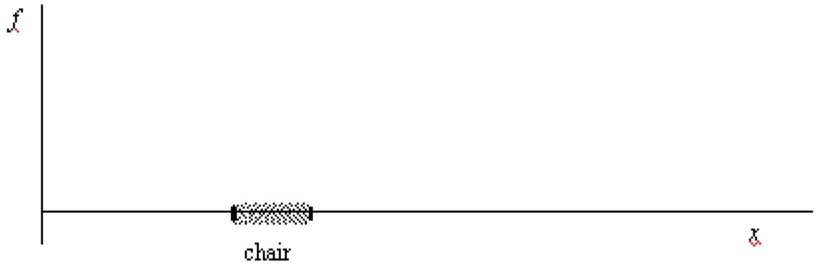
### Wave functions

Wave functions are functions from which one could extract information about particles such as momentum, energy, position, and other variables. Wave functions usually are complex, but the product $\Psi^*.\Psi$ is real. The product $\Psi^*(x,y,z).\Psi(x,y,z)$ provides the probability per unit volume, that is the *probability density* $f(x,y,z)$ to find an electron, proton, or other particle at a particular place. First we will teach the classical concept of probability.

[4] Remember Einstein's saying that *God does not play dice*.
[5] Although the word probability is used, from the context it will be clear that we usually mean probability *density*, per cm, or per $cm^2$, or per $cm^3$.

The teacher sketches figure 1 on the board and says:

*figure 1*




**Teacher:** *I have an x-axis* (delineates an x-axis in front of the room) *and on that x-axis I put a chair* (puts a chair). *I put a ballpoint under a piece of cloth "somewhere" on the chair* (puts cloth or handkerchief on chair and puts ballpoint somewhere under it). *I sketch the x-axis on the board and the gray area between the bars is the chair* (draws figure 1 on the board).
*The probability* (in this 1-dimensional case per cm) *to find the ballpoint is f(x).*
**Question 1**: *Sketch the probability f(x) as function of x. There are several acceptable solutions, so later compare with your neighbor*.

While the students are making their sketches, the teacher goes around the room, looks at student work, and asks an interpretation here and there: *What does your graph mean? Where is the greatest probability to find the ballpoint in your graph? How do you represent that in the graph?*

Some possible solutions are as follows (figure 2a,b):

figure 2a
Everywhere on the chair the probability is the same.

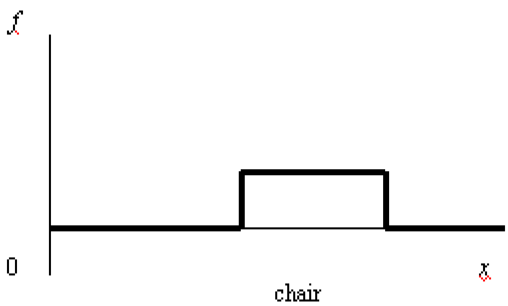
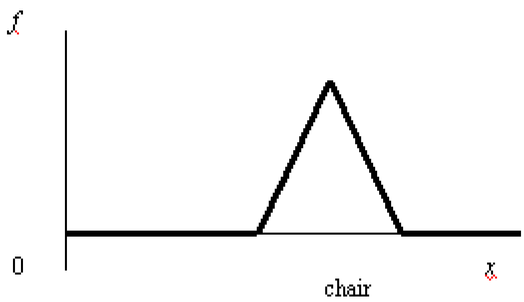


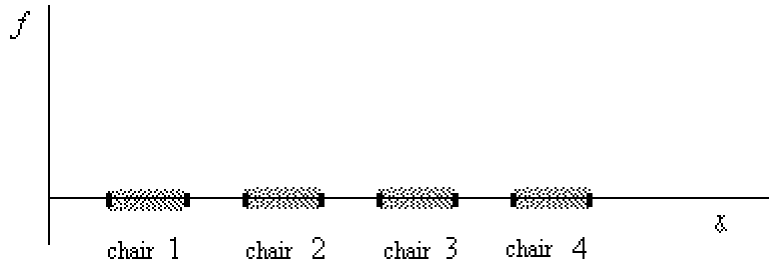

*figure 2b*
*In the middle of the chair the probability is greatest.*




In figure 2a the probability of finding the ballpoint anywhere on the chair is the same. In figure 2b it is more probable that the ballpoint is found in the middle of the chair. There could be a real physical reason for that, for example, when the chair is a little bit deeper in the middle as compared to the sides.

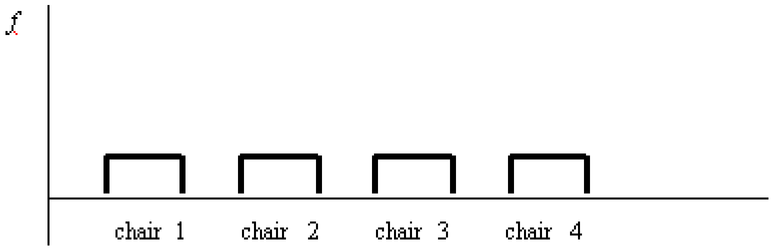
**Teacher:** *Now I have 4 chairs at some distance from each other* (we do then put 4 real chairs up front in addition to the chairs in the graph). *The ballpoint could be on any of these chairs and anywhere on their surface*. (We do put a ballpoint under a coat).
**Question 2:** *Sketch the probability*.

*figure 3*



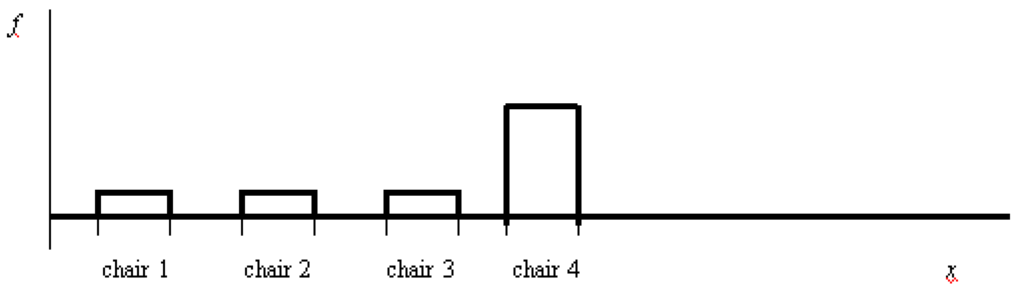

An answer can be found in figure 4. The sum of the areas in the graph should be 1 as that is the chance to find the ballpoint somewhere in the universe. Also here one could think of different solutions such as in figure 2b or an opposite solution where the probability of finding the ballpoint is greater on the sides of the chair (figure 7b).

*figure 4*




**Teacher:** *Suppose the guy who lost the ballpoint sat on chair 4 so that the chance to find my ballpoint there is greater than on chairs 1 – 3.*
**Question 3:** *How does the graph look? Sketch.*

*figure 5*
*The area under the graph is greater for the location of chair 4.*

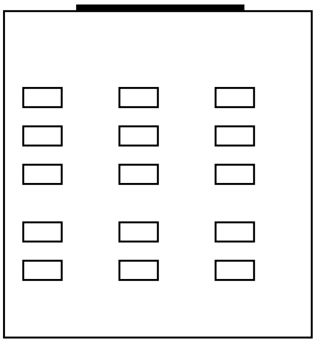



**Teacher:** *The area under the graph shows the probability to find the particle somewhere. The total probability should be 1.*
**Question 4:** *Should anything be adjusted in your graph of figure 5 in comparison to figure 3 to achieve a total area under the graph of 1?*

Now the teacher sketches on the board a map of the class and roughly indicates the location of tables and chairs.
**Teacher:** *There are places in the class where I often pass and there are places where I don't.*

**Question 5:** *Indicate through pencil marks where the probability[6] is high to find the teacher. Areas where that probability is zero remain white.*

*figure 6*
*A classroom situation where students can pencil in the probability for finding the teacher.*



Probably all benches will remain white as most teachers do not dance on top of student tables. Although, many teachers sometimes sit on one of the benches… In front of the room we usually find the most black area, but who knows, perhaps you are the kind of teacher who moves around a lot to unexpected parts of the classroom!

From here the jump to a probability density picture of an orbital is not that big (?) anymore.

If students need more exercise yet, one could still reverse the process and give graphs to the students and let them write a short interpretation (figures 7a, 7b)

*figure 7a*

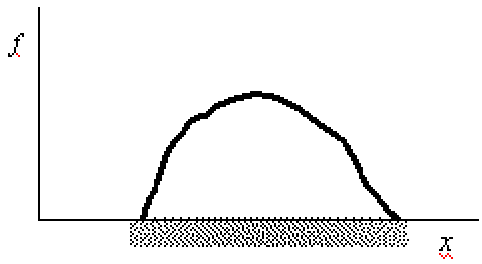
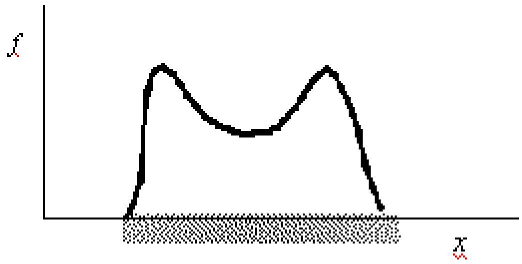



*figure 7b*

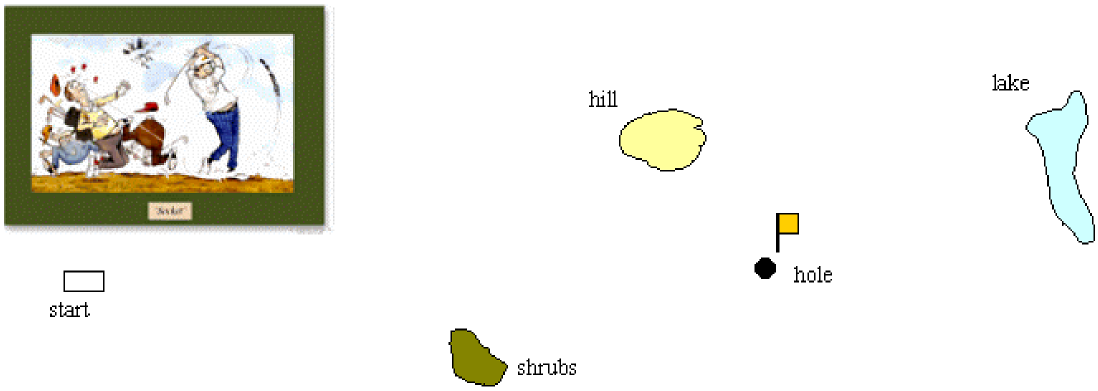



**Teacher:** *In figure 7a I have drawn the probability to find a ballpoint on the chair.*
**Question 6:** *Does this mean that the ballpoint is unlikely to be found on the side of the chair? Explain.*
**Question 7:** *Where on the chair is the probability greatest to find the ballpoint in figure 7b?*

Figure 8 shows a golf course. There is the start, the target hole, a little hill, a pond, and there are bush. We forgot the sand traps. We do an experiment in which 10,000 amateur golf players hit the ball at the start. Then we put a pencil dot where the ball lands. So we get a picture with 10,000 pencil dots. As a result we obtain a map showing light and dark places. The dark places have a high probability to find balls. Here one can let the students draw their golf ball probability plot. The resulting plot already looks like an orbital plot!

[6] In this case we graph probability density per $cm^2$. Wave functions in quantum physics usually concern probability density per volume as we are dealing with three-dimensional space.

*figure 8*




The next step is to start wondering about the difference between classical probabilities and the meaning of probability in quantum theory.

## Probability in quantum physics

Once students understand the work with probabilities, it is time to pay attention to the peculiarities, which are added by quantum physics. The normal, classical probabilities are often interpreted as lack of knowledge. The ballpoint on the chair has a very fixed location, except we do not know yet. Perhaps somebody, in this case the teacher, knows the exact location already. However, in quantum physics the probability is fundamental. The general interpretation is that the uncertainty cannot be reduced by more knowledge. Quantum objects such as electrons, protons, and other particles cannot be located with absolute certainty.

An important difference between the probabilities of finding golf balls somewhere in the field and the probability of electrons to hit a particular place on a screen, is that with electrons the probabilities have to do with wave functions. The behavior of waves leads to strange phenomena, which we do not encounter with golf balls. Waves can extinguish each other through interference. On a screen we could find interference patters with dark lines or rings. These are places that electrons cannot come *because they can get there in different ways* and interfere! Such patterns are also found when particles are shot at rather large time intervals, thus one by one. So even individual particles exhibit wave behavior, not just groups of particles. One possible point of view is to take distance from the idea that quantum physics describes individual systems or particles, but that the quantum theory only deals with ensembles of similar systems. In the words of Muller & Wiesner (2002):

> "As we have mentioned, a basic observation in an interference experiment with single photons is that the pattern on the screen builds up from the "hits" of single photons. It is legitimate to ask whether these positions are predetermined as in classical physics and can be predicted from the initial conditions. In this stage of the course, the students learn that one cannot predict the position of a single hit, but that it is nevertheless possible to make accurate predictions for the statistical distribution of *many* hits. This observation is generalized to the following important statement: *Quantum mechanics makes statistical predictions about the results of repeated measurements on an ensemble of identically prepared quantum objects.* This preliminary version of the probability interpretation is later, in the context of electrons, formulated more precisely in terms of the wave function." (Muller & Wiesner, 2002).

The question whether and how individual systems can be described disappears then into the background. That is unfortunate, but such is nature. What can be done is to exercise with differences between classic and quantum probabilities and particularly with interference phenomena.

# Example II: Applying conservation laws and symmetries to particle reactions

Lessons about elementary particles at the secondary school level can degenerate into listing a zoo of particles and reactions, resulting in disorganized and rather meaningless knowledge. A more powerful way is to focus on conservation laws, symmetries, and reaction diagrams. The conservation laws and symmetries provide generalizing power which enables the students to predict whether or not certain reactions are possible and to derive new reactions from given ones by applying the symmetries. For the theory we refer to the article Conservation Laws, Symmetries, and Elementary Particles which appeared in The Physics Teacher[10] in May 2005 and has been included in this booklet. Our students get the same text but then in Dutch. However, in practice the text functions as a back-up for study later and theory is taught through the following worksheet.

### Two lessons

In our Modern Physics project we spent ten 50-minute lessons on nuclear reactions and elementary particles. Only lessons 6 and 7 are relevant here. Earlier lessons are on recalling chemical reaction equations, extending the idea to nuclear reactions, energy and mass, binding energy, computations with mass deficits, and accelerators. Lesson 5 introduces the particles of the standard model (Tables 1 & 2). The exercises in lessons 6 and 7 are limited mainly to first generation particles except for the muon.

**Lesson 6**: In a short class discussion the teacher and students recall the conservation laws they have encountered so far (linear momentum, energy-mass, charge, and possibly angular momentum). Then the lesson proceeds in the following steps:

1. The *teacher* starts with the reaction equation: $\mathrm{p}^{+} + \mathrm{e}^{-} \rightarrow \mathrm{H}$ and gives an example of $\boldsymbol{C}$ symmetry by replacing particles with anti-particles: $\mathrm{p}^{-} + \mathrm{e}^{+} \rightarrow \overline{\mathrm{H}}$. The resulting anti-hydrogen was made at CERN, Geneva. So this reaction with anti-particles is indeed possible.
2. Then *students* answer exercises 1a and 1b from the worksheet (below) and perhaps an additional exercise added by the teacher. The teacher walks around and identifies any problems students may have with the exercise.
3. The *teacher* discusses the answers to 1a and 1b and perhaps one or two problems in understanding (s)he encountered when looking at the answers of students. Then the teacher gives an example of time symmetry using the ionization of hydrogen. $\mathrm{H} \rightarrow \mathrm{p}^{+} + \mathrm{e}^{-}$ . Reversing the arrow (time symmetry) also shows a possible reaction.
4. *Students* do exercise 1c and the teacher goes around and looks at answers.
5. The *teacher* discusses the answer to exercise 1c or skips that part altogether if everyone got it right. Then the teacher gives an example of the crossing operation. For example: $n + \nu_e \rightarrow p^{+} + e^{-}$. It turns out that we can move particles to the right or left of the arrow if we replace them by their antiparticles. The reaction $n \rightarrow p^{+} + e^{-} + \overline{\nu}_e$ is possible, but we are now dealing with an anti-neutrino. Whenever we apply the crossing operation to a valid and possible reaction, the particle has to be replaced by its anti-particle and we have another valid and possible reaction.
6. *Students* do exercises 1d and 1e and the teacher goes around to observe.
7. The *teacher* discusses 1d and 1e.
8. In the same way the class proceeds with exercises 2a-f.

**Lesson 7:**

9. Lesson 7 starts with an example of reaction diagrams (Figure 5).
   On the left of the vertex are reactants and on the right are products. An arrow to the right

stands for a particle and an arrow to the left stands for an anti-particle. For further details of these simplified Feynman diagrams we refer to our earlier article (Hoekzema et al, 2005).
Then exercises 3a-g are done with fast feedback, just like problems 1a-e and 2a-f in the previous lesson.
After every 1 or 2 exercises, the teacher interrupts, discusses the answers, and the class moves on to the following exercise.

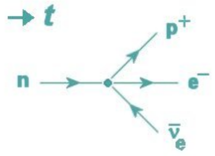



*Figure 5.* ***$\beta^-$* decay.**

10. Exercises 4 – 6 are done by students individually or in small groups at their own pace and no longer in fast feedback format, as these exercises take more thinking time. Thanks to the format of the worksheet, it is still possible for the teacher to very quickly assess the work of individual students and interact to find out the students' reasons for alternative answers and to engage in individual or small group discussions.

**Comments** (please read the worksheets first)
What students learn is the following: Given a reaction between certain particles, they can derive other possible reactions, and they do that by applying the conservation laws. Some might object that students just learn some tricks. We think that learning to apply conservation laws and symmetries to reactions is valuable and that understanding of symmetries at a much deeper level is not attainable at the secondary level and has to be postponed to university science programs.

## Literature

### Literature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# Worksheet on Symmetries and Reaction Diagrams with Answers

***1. β decay***

$$n \rightarrow p^{+} + e^{-} + \overline{\nu}_e \quad (1)$$

| **Exercise 1** | **Answers to exercise 1**[7] |
|---|---|
| a) Check for baryon, lepton, and charge conservation. | a) Baryon: 1 = 1<br>Lepton: 0 = 1 – 1<br>Charge: 0 = +1 –1 |
| b) Apply ***C*** symmetry to (1) and write the resulting equation | b) $\overline{n} \rightarrow p^{-} + e^{+} + \nu_e$<br>Please note that n consists of $udd$, while $\overline{n}$ consists of $\overline{udd}$ quarks, so neutron and anti-neutron are different[8]. |
| c) Apply ***T*** symmetry to (1) and write the resulting equation. | c) $p^{+} + e^{-} + \overline{\nu}_e \rightarrow n$ |
| d) Apply ***X***($\overline{\nu}_e$)-symmetry to (1) | d) $n + \nu_e \rightarrow p^{+} + e^{-}$ |
| e) Apply ***X***($e^{-}$)-symmetry to (1) | e) $n + e^{+} \rightarrow p^{+} + \overline{\nu}_e$ |

***2. Reactions with pions***

$$\pi^{-} + p^{+} \rightarrow \pi^{o} + n$$

| **Exercise 2** | **Answers to exercise 2** |
|---|---|
| a) Check for baryon and charge conservation. | a) Baryon: 0 + 1 = 0 + 1<br>Charge: -1 + 1 = 0 + 0 |
| b) Apply ***C*** symmetry to equation 2, where $\pi^{+}$ is taken as the anti-particle of $\pi^{-}$ and $\pi^{o}$ as the anti-particle of itself (see Table 2). | b) $\pi^{+} + p^{-} \rightarrow \pi^{o} + \overline{n}$ |
| c) Apply ***T*** symmetry to (2) | c) $\pi^{0} + n \rightarrow \pi^{-} + p^{+}$ |
| d) Apply ***X***(n) to (2) | d) $\pi^{-} + p^{+} + \overline{n} \rightarrow \pi^{0}$ |
| e) Why is the last reaction rather unlikely? | e) It is rather unlikely to find these three particles within 1 fm ($10^{-15}$ m) from each other. |
| f) The $\pi^{0}$ particle consists of an up quark and its anti-particle ($u\overline{u}$) or a down quark and its antiparticle ($d\overline{d}$). Will the particle last long? Explain. | f) Annihilation can take place between $u$ and $\overline{u}$ or $d$ and $\overline{d}$ but not between quarks of different flavour such as $u$ and $\overline{d}$ and $\overline{u}$ and $d$. |

[7] The student version should have a blank second column!

[8] Strictly speaking, all anti-particles should be written with a bar ($\overline{e}^{+}, \overline{\nu}_e, \overline{n}$); however, it is a custom to just write $p^{-}$ and $e^{+}$ rather than $\overline{p}^{-}$ and $\overline{e}^{+}$.

3. ***Muon decay***

The reaction for muon decay is:

$$\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu \qquad (3)$$

The reaction diagram[9] is shown in Figure 6.

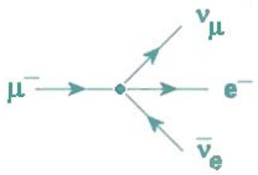
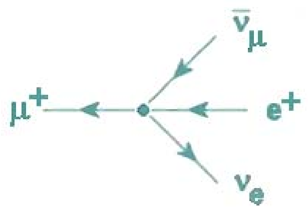
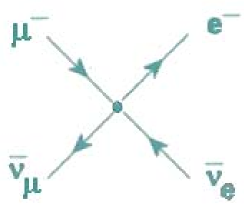
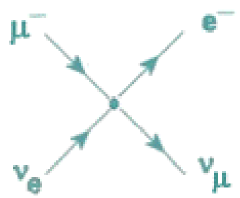



**Figure 6.** Muon decay.

| **Exercise 3** | **Answers to exercise 3** |
|---|---|
| a) Check for lepton conservation<br><br>b) Apply ***C*** symmetry to (3)<br><br>c) Apply ***X***($\nu_\mu$) to (3)<br><br>d) Apply ***X***($\overline{\nu}_e$) to (3) | a) $\mu$ leptons: +1 = +1<br>e leptons: 0 = +1 –1<br>b) $\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$<br>c) $\mu^- + \overline{\nu}_\mu \rightarrow e^- + \overline{\nu}_e$<br>d) $\mu^- + \nu_e \rightarrow e^- + \nu_\mu$ |
| e) Draw the reaction diagram of 3b | e) Antimuon decay.<br> |
| f) Draw the reaction diagram of 3c | f) Reaction of muon with antimuonneutrino.<br> |
| g) Draw the reaction diagram of 3d | g) Reaction of muon with electronneutrino.<br> |

[9] At this point the teacher will introduce reaction diagrams and then the students continue with questions 3a-3g using the fast feedback method. Thus the teacher discusses the answer to 3a before students proceed to 3b, etc.

## 4. *Once again beta decay*[10]

We return to β decay:

$$n \rightarrow p^{+} + e^{-} + \overline{\nu}_e \qquad (4)$$

| **Exercise 4** | **Answers to exercise 4** |
|---|---|
| a) Use symmetries to derive an equation for beta decay, which results in the emission of a positron[11]. | a) $\boldsymbol{X}(e^{-})$ would produce a positron on the left of the arrow. Then we apply T symmetry and reverse the arrow: $p^{+} + \overline{\nu}_e \rightarrow n + e^{+}$ |
| b) Show that it is not possible to derive a reaction from equation (4) using symmetries in which, amongst others, a positron is produced from a neutron. | b) Using crossing symmetry with the particles of equation (4) only, we would always end up with a positron and a neutron on the same side of the arrow. On the opposite side $e^{+}$ would have to be replaced by $e^{-}$. |
| c) Use the symmetries and try to derive an equation in which an electron is absorbed into the nucleus. (In nature this can happen spontaneously in nuclei with high Z. It can also be contrived by shooting electrons at nuclei. | c) As *input* we need an electron. So we apply Time symmetry on (4) and then move the anti-neutrino to the right using crossing: $p^{+} + e^{-} \rightarrow n + \nu_e$ We can also get this by applying time symmetry to the answer to 1d. |
| d) Look at the equations once again. Which process could be used to detect electron neutrinos and electron anti-neutrinos[12]? | d) We can detect electron neutrinos through collisions with neutrons $n + \nu_e \rightarrow p^{+} + e^{-}$ and anti-neutrinos through collisions with protons: $p^{+} + \overline{\nu}_e \rightarrow n + e^{+}$ |
| e) The reaction in equation (4) can take place in a free neutron, but it is much more likely to occur in a neutron which is part of a nucleus such as ${}^{37}_{17}Cl$. Write this reaction for chlorine-37. | e) ${}^{37}_{17}Cl \rightarrow {}^{37}_{18}Ar + e^{-} + \overline{\nu}_e$ |
| f) By crossing the reaction in chlorine-37 we can get a reaction, which makes it possible to discover neutrinos when they collide with a chlorine nucleus. Write the reaction and add a reaction diagram[13]. | f) ${}^{37}_{17}Cl + \nu_e \rightarrow {}^{37}_{18}Ar + e^{-}$ or $n + \nu_e \rightarrow p^{+} + e^{-}$ |

[10] From this point on students work either in small groups or alone at their own pace and no longer with fast feedback. However, the format of the worksheet still allows the teacher to supply quick individual feedback.
[11] With questions 4a and 4b things get interesting. We can derive all forms of beta decay from just one equation (1). We can also immediately judge whether a certain variation is possible or not. For example, we can immediately judge whether a certain variation is possible or not. So we can predict that absorbing an electron in a heavy nucleus is possible. However, such an electron cannot remain an electron, as its typical wavelength ($10^{-10}$ m) would not fit the nucleus ($10^{-15}$ m).
[12] This question once again shows how the use of symmetries can lead to important predictions. It is indeed possible to detect neutrinos using these reactions.
[13] The reaction with chlorine was used by Nobel laureate Davis (2002) to detect and count neutrinos emitted by the Sun.

***5. Collisions***

Check whether the following reactions are possible or not and indicate why.

| **Exercise 5** | **Answers to exercise 5** |
|---|---|
| a) $\pi^+ + p^+ \rightarrow p^+ + p^+ + \overline{n}$<br>b) $p^+ + p^+ \rightarrow p^+ + p^+ + n$ | a) Baryon conservation is okay:<br>0 + 1 = 1 + 1 –1.<br>Also charge conservation is okay.<br>b) No baryon conservation as: $2 \neq 3$ |

***6. What kind of particles?***

A reaction is as follows:

$$p^+ + p^+ \rightarrow p^+ + p^+ + X$$

X is an unknown particle.

| **Exercise 6** | **Answers to exercise 6** |
|---|---|
| a) Is it a meson or a baryon? Why? | a) X cannot be a baryon or anti-baryon, as the baryon number would not be conserved. It could be a neutral meson. |
| b) Does X have charge or not? Why? | b) X cannot have charge, as there would be no charge conservation. |
| c) Can X be a lepton? | c) If X would be a lepton, there would not be conservation of lepton number. |
| d) Answer a), b), and c) in case two particles are formed (X and Y). | d) A baryon and an anti-baryon would be possible if X and Y would both be neutral or would have opposite charge. Leptons would be possible, but then it would have to be a lepton and its anti-lepton. |

Table 1: Elementary Particles

| Elementary Particles: Fermions | | | | | | | |
|---|---|---|---|---|---|---|---|
| Quarks[1] | | | | Leptons[2] | | | |
| Generation | Particle/flavor | Mass (GeV/$c^2$) | Charge (e) | Generation | Particle/flavor | Mass (GeV/$c^2$) | Charge (e) |
| 1 | u up quark | 0.003 | 2/3 | 1 | $\nu_e$ electron neutrino | <1. $10^{-5}$ | 0 |
| | d down quark | 0.006 | -1/3 | | $e^-$ electron | 0.000511 | -1 |
| 2 | c charm quark | 1.3 | 2/3 | 2 | $\nu_\mu$ muon neutrino | <0.0002 | 0 |
| | s strange quark | 0.1 | -1/3 | | $\mu^-$ muon | 0.106 | -1 |
| 3 | t top quark | 175 | 2/3 | 3 | $\nu_\tau$ tau neutrino | <0.02 | 0 |
| | b bottom quark | 4.3 | -1/3 | | $\tau$ tau | 1.7771 | -1 |
| | | | | | | | |
| **Elementary Particles: Bosons** | | | | | | | |
| Strong interaction | | | | | Electro-weak interaction | | |
| | g gluon | 0 | 0 | | $\gamma$ photon | 0 | 0 |
| | | | | | $W^-$ W minus boson | 80.4 | -1 |
| Gravitation | | | | | $W^+$ W plus boson | 80.4 | +1 |
| | graviton (hypothetical) | | | | $Z^0$ Z boson | 91.2 | 0 |

a. For every quark there is an anti-quark with the same mass, opposite charge and baryon number –1.

b. For every lepton there is an anti-lepton with the same mass, opposite charge, and lepton number –1.

c. Conservation of lepton number is considered separate for electron and electron neutrino, muon and muon neutrino, and tau particle and tau neutrino.

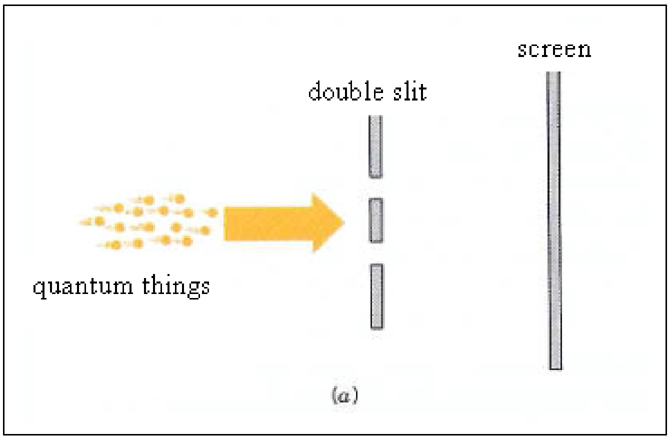
Table 2: Some compound particles

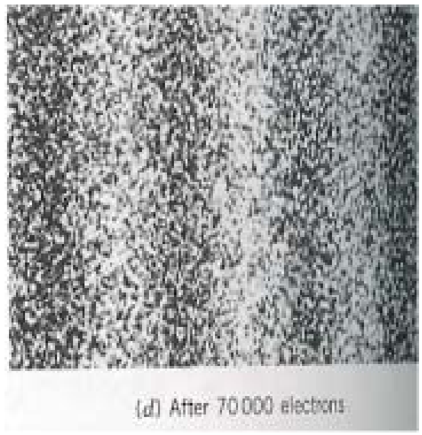
| Several compound particles | | |
|---|---|---|
| particle | composition | Baryon number |
| $p^+$ proton | uud | 1 |
| $p^-$ anti-proton | $\overline{u}\overline{u}\overline{d}$ | -1 |
| n neutron | udd | 1 |
| $\overline{n}$ anti-neutron | $\overline{u}\overline{d}\overline{d}$ | -1 |
| $\pi^-$ pi minus meson | $\overline{u}d$ | 0 |
| $\pi^+$ pi plus meson | $u\overline{d}$ | 0 |
| $\pi^0$ pi meson | $u\overline{u}$ of $d\overline{d}$ | 0 |

# Example III: Worksheet Particles and Waves

In figure 1a a beam of particles, or waves, or in any case ‘quantum things’, is fired at a double slit screen. Figure 1d shows a possible result: a screen hit by 70 000 electrons.

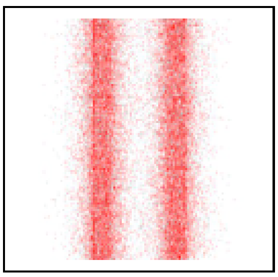
*figure 1*
*Double slit experiment.*



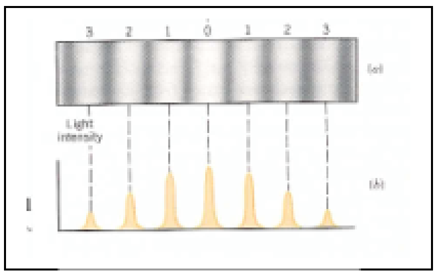




What appears on the screen depends among other things on the distance between the slits and the type of objects being fired. The pattern in figure 2A was made by firing small bullets through a two-slit metal plate; figure 2B is a result from Young’s experiment, with light falling through a two-slit diaphragm.

*figure 2*
*Particles and waves*



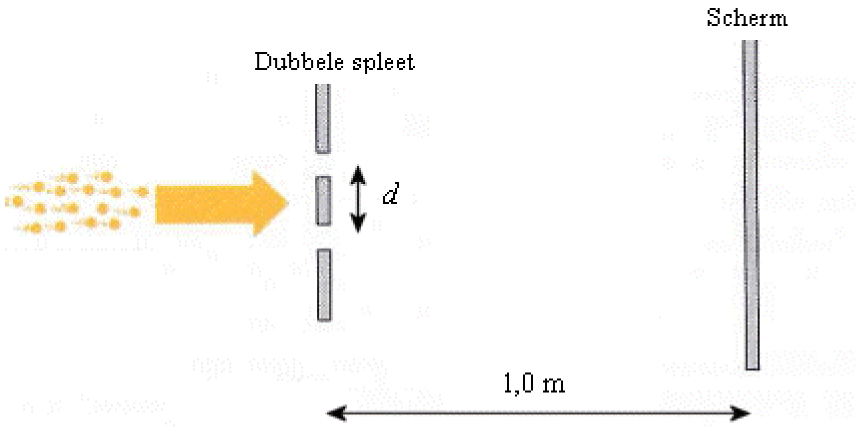
2A: bullets



2B: waves

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>1. Is figure 1d more like 2A or like 2B?<br>Explain your answer.<br><br>2. Which of the two figures 2A of 2B shows interference?<br>Circle one of the answers a)...d) and explain.</td><td><br><br><br>a) A    c) A en B<br>b) B    d) None</td></tr>
<tr><td colspan="2">In the following questions, different objects are fired at slits in different ways. The distance d between the slits is called big if it is large compared to the De Broglie-wavelength and small if it is of the same order of magnitude.<br>Explain in each case whether the screen image is more like 2A or like 2B.</td></tr>
<tr><td>3. A beam of red light is pointed at two slits with small d.<br><br>4. Beam intensity is decreased until photons pass the slits one by one. A screen pattern is obtained by recording single photon hits with a very sensitive CCD-plate, and waiting a few days.<br><br>5. A beam of electrons is pointed at two slits with small d.<br><br>6. Beam intensity is decreased until electrons pass the slits one by one. A screen pattern is obtained by recording single electron hits.<br><br>7. An electron beam is pointed at two slits with large d.<br><br>8. A beam of red light is pointed at two slits with large d.</td><td></td></tr>
<tr><td>9. Summarize your answers.</td><td></td></tr>
</table>

With the following questions we aim to get a clearer picture of what it means for $d$ to be small or large. Each time, the distance between diaphragm and screen is 1,0 m, but wavelength and slit-distance $d$ will vary.




Calculate the distance between the $0^{th}$- and $1^{th}$-order maximum on the screen.

10. A beam of red light hits a diaphragm with $d$ = 1 mm.

11. A beam of electrons with an energy of 10 eV hits a diaphragm with $d$ = 1 mm.

12. A beam of electrons with an energy of 10 keV hits a diaphragm with $d$ = 1 mm.

13. A beam of electrons with an energy of 10 keV hits a diaphragm with $d$ = 1 nm.

14. A beam of red light hits a diaphragm with $d$ = 1 nm.

# Conservation Laws, Symmetries, and Elementary Particles

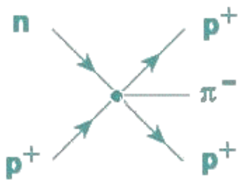
**See our article in the May 2005 Issue of The Physics Teacher**

# Examples of a Modern Physics Exam Problems

## Pions

Pions can be produced by accelerating protons and letting them collide with a target plate. The following reaction can occur:

**A:** $p^+ + n \rightarrow 2p^+ + \pi^-$




The proton must have a minimal kinetic energy for this process to occur.

3p **12** □ Compute this minimal energy in MeV.

Next to the negative pion there are two other kinds, the $\pi^+$ and the $\pi^0$. The pions consist of a quark and an anti-quark of the first generation: u and d, $\bar{u}$ and $\bar{d}$.

2p **13** □ Determine the quark composition of the negative pion.

The pion that is formed in reaction A can collide with a proton and result in reaction B.

**B:** $\pi^- + p^+ \rightarrow n + \pi^0$

The neutral pion is a remarkable particle which is $u\bar{u}$ but also $d\bar{d}$. The pion continuously goes from $u\bar{u}$ to $d\bar{d}$ and vice-versa.

2p **14** □ Explain why the neutral pion will not exist very long.

Using symmetry transformations, other reactions can be predicted. Compared to reaction B, the two pions can be crossed $X(\pi^-, \pi^0)$, and on the resulting reaction we can apply time reversal $T$.

4p **15** □ Draw the diagram of the resulting reaction and explain your answer.

**End**

# Pions: Correction Model

3p **1** □ Answer: 138 MeV *points*

Example of computation:
$\Delta m = 2\,m_{p} + m_{\pi-} - m_{p} - m_{n} = m_{p} + m_{\pi-} - m_{n} = 938 + 140 - 940 = 138\ \text{MeV}/c^2$
Then $\Delta E = 138$ MeV

- Writing down the mass equation and substituting the proper masses. *[1]*
- Realisation that the mass of $\pi^-$ equals that of $\pi^+$ *[1]*
- Completion of the computation *[1]*

2p **2** □ Answer: $\bar{u}d$

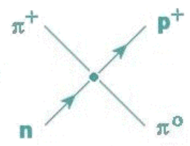
This is the only possible combination which results in charge –e

- Use of quark charges (from a booklet with physics constants) *[1]*
- Conclusion *[1]*

2p **3** □ Example of an answer: Whether $\pi^0$ exists of an $u\bar{u}$ or $d\bar{d}$ or another mixture, it is always a quark with its own anti-quark and thus can easily annihilate.

- Mention of annihilation *[1]*
- Explanation and conclusion *[1]*

4p **4** □ Answer:




$$n + \pi^+ \rightarrow p^+ + \pi^0$$

- At least one symmetry operation has been carried out correctly.
- Reaction is correct as equation or as diagram *[1]*
- Arrows of p and n have the proper direction *[1]*
- No arrow for $\pi^0$ but only a line without arrow (as it cannot be classified as particle or anti-particle) *[1]*

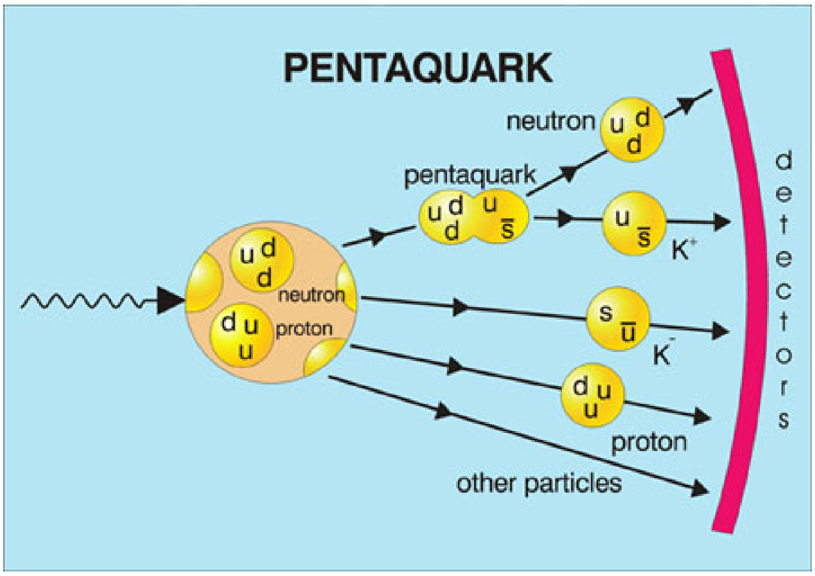
Note: is students write an arrow for $\pi^+$, no points will be deducted. Note that mesons cannot be arrows as none of them can be clearly classified as particle of anti-particle as all mesons consist of a quark and an anti-quark.

# Pentaquark

Read the newspaper article below which has been translated from the Dutch Science Supplement of the NRC-Handelsblad.

## Physicists discover particle consisting of five quarks

*figure 1*




Two teams of physicists have each discovered a particle that is made of five quarks, a so-called pentaquark. A Japanese group of the University of Osako saw the pentaquark first. The Japanese directed a powerful bundle of gamma rays (up to 2.4 GeV) at a plastic target. Collisions between the gamma particle and a neutron in a carbon atom produced the pentaquark. The mass was determined to be 1.54 GeV/$c^2$. The pentaquark turns out to consist of two down-quarks, two up-quarks and an anti-strange-quark. It decays very quickly into a neutron and a positive K-meson. The Japanese discovery has already been confirmed by an American group of scientists of the University of Ohio. It is still unclear whether the pentaquark consists of five strongly bonded quarks or that it is a kind of molecule consisting of a neutron and a positive K-meson.

The newspaper article suggests that the following reaction took place:

$$\gamma + n \rightarrow \text{pentaquark}$$

**2p** 6 □ Explain with a conservation law that this reaction would have been impossible..

A reaction that could have taken place is the following:

$$\gamma + n \rightarrow \text{pentaquark} + K^-$$

**3p** 7 □ Calculate the minimum energy in MeV of the gammaphoton to make this reaction possible. Assume that the velocity of the neutron is zero.

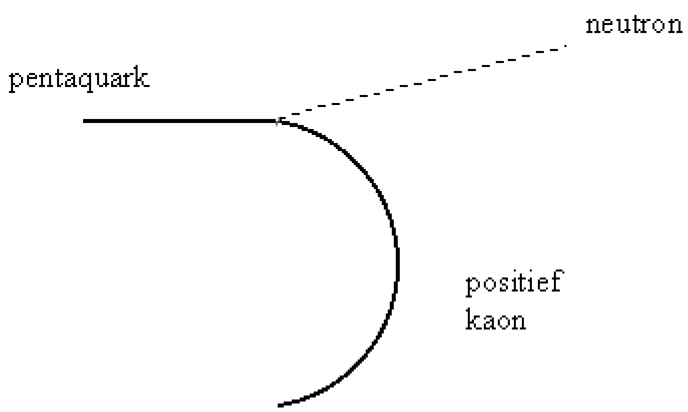
**4p** 8 □ Draw the reaction diagram of this reaction. Draw the K particle without an arrow.

Other ways to make a pentaquark can be found through the application of symmetry operations on the reaction.

**4p** 9 □ Show two more ways to make a pentaquark and indicate which symmetries you used to find these reactions.

The mass of the pentaquark was determined through by letting the decay of the pentaquark take place in a homogeneous magnetic field. Figure 2 shows this decay as it was observed by a detector. The magnetic field is perpendicular to the plane of the drawing.

*figure 2*




To determine the mass of the pentaquark, one should know –amongst others- the momentum of the positive kaon. The momentum can be determined from figure 2.

**3p** 10 □ Explain how the momentum of the positive kaon can be determined from figure 2. First derive an expression for the relationship between momentum and the radius of the orbit. (N.B. You are not asked to execute the determination, only to explain how it could be done.)

## Pentaquark: Correction Model

2p **1** □ Example of an answer:
The reaction is impossible due to charge conservation. On the left total charge is zero and on the right +e (a positive Kaon in the "molekuul" if you see the pentaquark as a "molecule")

- Correct determination of charge on the left and on the right *[1]*
- Mention of charge conservation *[1]*

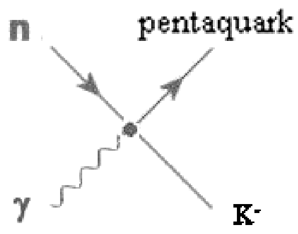
*Comment:an answer based on conservation of strangeness is correct as well*

3p **2** □ Answer: $1{,}09 \cdot 10^3$ MeV

Example of a computation:
$\Delta m = 1{,}54 \cdot 10^3 + 494 - 940 = 1{,}09 \cdot 10^3$ MeV/$c^2$. The photon energy should be at least $1{,}09 \cdot 10^3$ MeV.

- Use of $m$(pentaquark) and getting the correct other masses (from a book with tables) *[1]*
- Understanding that $\Delta m = m(\text{P}) + m(\text{K}) - m(\text{n})$ *[1]*
- Completing the computation *[1]*

4p **3** □ Answer:




- Particles left *[1]*
- Particles right *[1]*
- Arrows for neutron and for pentaquark *[1]*
- No arrow for the photon *[1]*

4p **4** □ Example of an answer:

$$\text{original: } \gamma + n \rightarrow P + K^-$$

$$\text{X}(K^-) \Rightarrow \gamma + n + K^+ \rightarrow P$$

$$\text{X}(\gamma) \quad \Rightarrow n + K^+ \rightarrow P + \gamma$$

- Per correct reaction *[1]*
- Per symmetry-operation *[1]*

*Comment:* $n + K^+ \rightarrow P$ , by applying X($K^-$) and leaving out γ is okay. The γ is not necessary if n or $K^+$ have sufficient energy.

3p **5** □ Example of an answer:
The momentum of the kaon is determined from the radius of the magnetic field through $p = BeR$.
So you have to measure the radius of the trajectory. One also has to know the strength of the field.

- Understanding that $F_z = F_{mpz}$
- $p = BeR$ *[1]*
- Understanding that one has to measure $B$ and $R$ *[1]*

*[1]*